

# What is Dark Matter's Impact on Structure Formation and Evolution?

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## I. Introduction

Dark matter is one of the most profound mysteries in cosmology. While it neither emits nor absorbs light, its gravitational influences are essential in explaining the formation and evolution of our universe. From the rotation speeds of galaxies, to the minor anisotropies in the cosmic microwave background, observational evidence strongly supports the existence of this massive, invisible component of our universe that largely outweighs ordinary baryonic matter. Within the  $\Lambda$ CDM model, dark matter is the scaffolding on which galaxies and large-scale structures are built, while dark energy drives the accelerated expansion of the universe. Exploring and understanding dark matter is essential to uncovering the cosmos we know today, as well as interpreting fundamental physics that govern the universe.

This paper will begin by examining dark matter's role in the  $\Lambda$ CDM model, in particular its relations to the cosmic web structure and energy densities since the early universe. From there, we will go through the key observational pieces of evidence that support dark matter's existence. This includes galaxy rotational curves, gravitational lensing, the Bullet Cluster, cosmic microwave background radiation, and structure formation, all of which provide strong reasoning for non-baryonic matter in our universe. Lastly, this paper will explore different dark matter particle candidates that have been considered over the years.

## II. Lambda CDM

### A. *The Cosmic Web*

The cosmic web, or the large-scale structure of the universe, consists of a vast framework of filaments, nodes, and voids. Filaments are thread-like structures made of dark matter along which galaxies cluster.<sup>1</sup> They channel gas and galaxies towards more concentrated areas called nodes, which host massive galaxy clusters.<sup>2</sup> The vast, underdense empty regions that largely make up the cosmic web are voids.<sup>2</sup>

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<sup>1</sup> Freese, 2008

<sup>2</sup> Stapelberg, 2022

This structure is based on a scaffolding provided by the gravitational influence of dark matter. Slight dark matter density fluctuations during the Big Bang grew as the universe expanded, forming the concentrated filaments and dense nodes and leaving the empty voids in between them.<sup>1</sup> Galaxies form and evolve along these dark matter filaments, and we are often able to detect baryonic gas in these areas as well. This is why there is often underlying dark matter in galaxy-dense regions.<sup>3</sup>

The  $\Lambda$ CDM model naturally incorporates the cosmic web and its behaviors. For instance, the web-like formation of filaments, which is a result of dark matter density fluctuations in the cosmic microwave background, is largely driven by the abundance of Cold Dark Matter. The structure formations simulated by the  $\Lambda$ CDM model also produce important features such as the size of cosmic voids and the concentration of galaxy clusters at nodes. Moreover, it predicts how dark energy ( $\Lambda$ ) drives the acceleration and expansion of the universe, additionally impacting the structure and definition of the cosmic web.

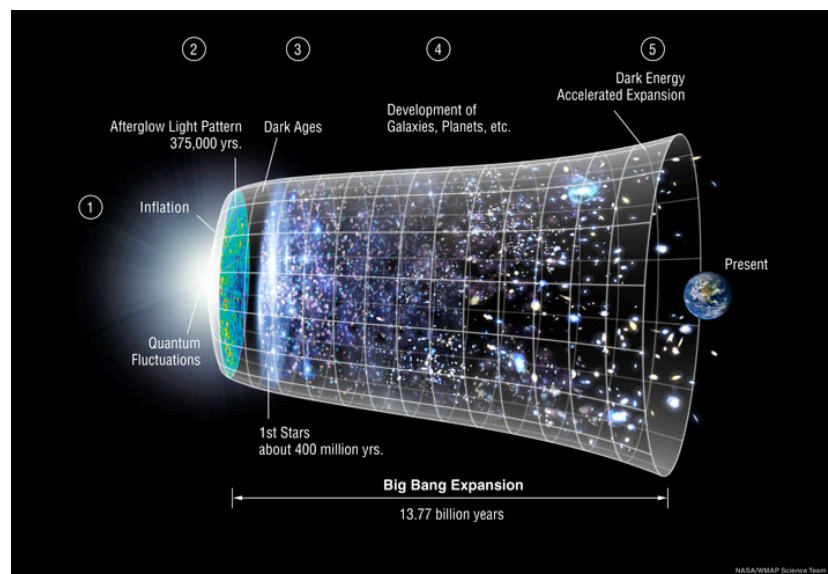


Figure 1. Image from NASA/ LAMBDA Archive / WMAP Science Team.

The  $\Lambda$ CDM model describes the universe's expansion history from the Big Bang to the present. Figure 1 displays this timeline, beginning with inflation and quantum fluctuations, followed by the release of the cosmic microwave background, the "dark ages," and the formation of the first stars around 375,000 years ago. As large-scale structures began to form in the matter-dominated universe, the expansion was overtaken by dark energy, leading to the current epoch of accelerated expansion. This progression captures these central features of the  $\Lambda$ CDM model.

<sup>3</sup> Vankov *et al*, 2024

## B. Energy Densities & Scale Factor

Within the  $\Lambda$ CDM model, the expansion of the universe has been driven by the relative energy densities of radiation, matter (both baryonic and dark matter), and dark energy dilutes or evolves as the universe stretches<sup>4</sup>. The scale factor,  $a(t)$ , is a dimensionless function of cosmic time that measures how distances in the universe expand relative to their value today<sup>4</sup>.

When  $a = 1$ , we are in the present day.<sup>5</sup> When  $a < 1$ , we are looking back in time, when the universe was smaller and denser.<sup>5</sup> When  $a > 1$ , we are projecting forward into a larger, more expanded universe.<sup>5</sup>

These scaling behaviors are displayed in the Friedmann Equations:

$$H^2(a) = H_0^2 [\Omega_r a^{-4} + \Omega_m a^{-3} + \Omega_\Lambda]$$

Where  $H(a)$  is the Hubble parameter at a scale factor  $a$ ,  $H_0$  is its present value, and  $\Omega_i$  are the fractional energy densities today.<sup>5</sup> The  $\Lambda$ CDM model matches these observed transitions between epochs, which is one of its biggest successes.

The different scaling with  $a$  represent fundamental properties of the expansion of the Universe throughout its history.

In the radiation-dominated era, relativistic particles controlled the expansion, shaping the cosmic microwave background and the slight density perturbations.<sup>4</sup> Radiation energy density scales as  $\rho_r \propto a^{-4}$ . This is because the number density of photons and other relativistic particles decreases with the expansion as  $a^{-3}$ , while their wavelengths stretch due to redshift, introducing the additional factor of  $a^{-1}$ .<sup>4</sup> In other words, radiation thins out the fastest due to its energy being tied to both the stretching of space and the redshifting of photons.

As the universe continued to expand, matter came to dominate. This allowed dark matter to drive the formation of galaxies and clusters. Matter energy density is dominated by baryons and cold dark matter.<sup>5</sup> Therefore, it scales as  $\rho_m \propto a^{-3}$ , reflecting only the dilution from volume increase, making it dilute more slowly.<sup>5</sup>

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<sup>4</sup> Akarsu, Uzun, 2023

<sup>5</sup> Greben, 2025

Eventually, dark energy began to dominate, accelerating the expansion of the universe. Dark energy, which is described by the cosmological constant  $\Lambda$ , remains constant, as the name suggests.  $\rho_\Lambda$  is independent from the scale factor.<sup>5</sup>

So, the scale factor acts as a ruler that allows us to measure the "stretching" of the universe as time progresses.

### III. Observational Pieces of Evidence

Although dark matter has never been directly detected, a large variety of observational evidence consistently points to the presence throughout our universe. Phenomena such as the flat rotation of galaxies, the gravitational lensing of unseen mass, and the separation of baryonic and non-baryonic matter in galaxy cluster collisions such as the Bullet Cluster cannot be explained by ordinary matter alone. The CMB and large-scale structure further show that non-luminous matter is essential for validating the events that occur throughout the cosmos. Together, these independent pieces of evidence provide a strong case for the existence of dark matter.

#### A. Rotation Curves

The measurements of rotation curves within galaxies are one of the strongest pieces of observational evidence supporting dark matter's existence. Rotation curves are graphs that display how the orbital speed of objects varies with their distance from the center of a rotating object, such as a galaxy or star system.<sup>1</sup> These curves are used to study the distribution of mass within these objects.

Kepler's Third Law, the law of orbital motion ( $P^2 = a^3$ ), states that rotation speed should drop with distance from the center, as in the solar system.<sup>6</sup> If galaxies consisted of only visible matter, they should follow this principle, but observations show that flat or rising rotation curves. In other words, stars towards the outskirts of galaxies move just as fast or even faster than those towards the center.<sup>1</sup> If only luminous matter is present, this contradicts Kepler's law. Therefore, the obvious solution is the existence of some sort of non-baryonic matter that explains the extra mass.

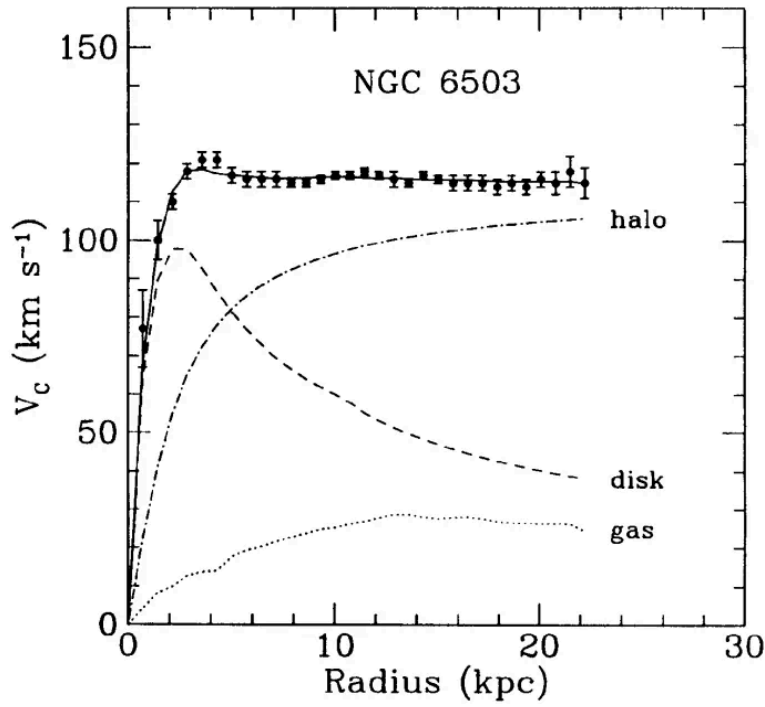
The virial theorem provides another way to describe this discrepancy.

$$2\langle T \rangle + \langle U \rangle = 0$$

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<sup>6</sup> Sands *et al*, 2024

This theorem relates average kinetic energy  $\langle T \rangle$  to average potential energy  $\langle U \rangle$  of a system in steady-state equilibrium.<sup>6</sup> The kinetic energy depends on the orbital velocities of the



stars, while potential energy is dependent on the total mass, allowing the calculation of the total dynamical mass of the galaxy. When we compare this number to the mass inferred from stars and gas, the virial estimate is significantly larger. This further confirms the existence of dark matter.

Figure 2. Image from Freese, 2008.

Figure 2 displays rotation curve data for galaxy NGC 6503. The black squares with error bars represent the observed rotational velocities, while the dashed line represents the rotational velocity contribution from the luminous matter in the galactic disks. The total contributions from the rotational velocities of the disk and gas do not match the observed flat rotation curve of the galaxy at larger radii. To account for this discrepancy, the dark matter halo is introduced. When combined with the disk and gas, the rotational velocities match the observed flat rotation curve of the galaxy.

Dark matter contains unseen mass that does not emit light but has gravitational effects.<sup>1</sup> Adding enormous dark matter halos to the galaxies matches our observational data much closer, as the baryonic data alone cannot account for the mass.

There are certain limitations to utilizing rotation curves in order to detect dark matter. Firstly, we can only look out as far as there is light being emitted, or neutral hydrogen (wavelength is 21 cm).<sup>1</sup> Additionally, one may be able to see the beginnings of dark matter halos, but we cannot trace where most of the dark matter is.

## **B. Gravitational Lensing**

According to Einstein's Theory of General Relativity, mass bends, or lenses, light<sup>7</sup>. This gravitational effect can be used in order to confirm the existence of more mass that doesn't emit light.

Both strong and weak gravitational lensing provide solid observational evidence for dark matter's existence. Strong lensing results in clear visual distortions to the background galaxies.<sup>7</sup> For example, Abell 1689 is a massive galaxy cluster known for its strong gravitational lensing.<sup>8</sup> Astronomers were able to use this lensing to infer the overall dark matter distribution.

Weak gravitational lensing causes small distortions to the background galaxies.<sup>9</sup> By taking a large sample of lensing in background galaxies, scientists are able to produce accurate calculations regarding the dark matter halos surrounding these galaxies. A study conducted about the flat rotation curves of galaxies utilized weak lensing to derive the circular velocity curves and Tully-Fisher relation (links spiral galaxies luminosity to its rotation velocity).<sup>9</sup>

Refer to the figure in section 3A, Rotation Curves, which displays the galactic rotation curve for spiral galaxy NGC 6503. It illustrates the relationship between the rotational velocity of matter and its distance from the center of the galaxy. The significant difference between the observed rotation curve (black boxes with error lines) and the contributions of visible matter (disk and gas) strongly suggest the presence of a large amount of dark matter in the galactic halo. It is necessary to account for the flat rotation curve at large distances.<sup>1</sup>

Mass-to-light ratios derived from gravitational lensing data match dark matter predictions from other cosmic structures, such as the  $\Lambda$ CDM model and the cosmic web.

## **C. Bullet Cluster**

The Bullet Cluster is one of the strongest pieces of observational evidence for dark matter. It is named for the system formed when a smaller subcluster passed through a larger

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<sup>7</sup> Vegetti *et al*, 2023

<sup>8</sup> Taylor *et al*, 1998

<sup>9</sup> Huterer, 2010

galaxy cluster.<sup>10</sup> The Bullet Cluster shows a separation between gravitational mass and mass from hot gas. Due to gravitational lensing, we can see mass centered where there is little visible matter.<sup>10</sup>



Figure 3. Image from X-ray: NASA/ CXC/ CfA/ M.Markevitch.

While the two clusters were merging, due to electromagnetic forces and friction, hot gas was slowed and merged into a single region at the collision point. However, dark matter, which was unaffected by these forces, passed through and kept its trajectory. This resulted in two distinct mass peaks - one for visible matter and one for dark matter.

As discovered through gravitational lensing, dark matter (the blue regions) form separate clumps from baryonic matter (pink regions).<sup>10</sup> This provides further proof for the existence of dark matter within galaxies and galaxy clusters.

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<sup>10</sup> Robertson *et al*, 2016

The majority of a galaxy cluster's mass is composed of dark matter, which essentially acts as the gravitational "glue" holding the entire cluster together. The hot gas fills the spaces in between the galaxies, making up about 10-15% of the total cluster mass.

#### D. CMB

Recall section 2A, Standard Model of Cosmology ( $\Lambda$ CDM), the universe started as a singularity and underwent rapid exponential expansion (cosmic inflation). Tiny quantum fluctuations in the early universe were exacerbated, and over 14 billion years grew into the large-scale structures we see today.<sup>3</sup> Evidence of this transformation is seen in the cosmic microwave background (CMB) - the relic radiation left over from the universe's birth. In the dense early universe, matter existed as a plasma where protons and electrons were tightly coupled.<sup>11</sup> As the universe expanded and cooled, photons decoupled from matter and light was released for the first time. This "snapshot" of the infant universe is what we observe as CMB today.<sup>12</sup>

The CMB is not perfectly uniform; it contains slight anisotropies, which reveal the matter and energy distribution in the early universe.<sup>11</sup> Crucially, the observed pattern of anisotropies cannot be explained by baryonic matter alone.

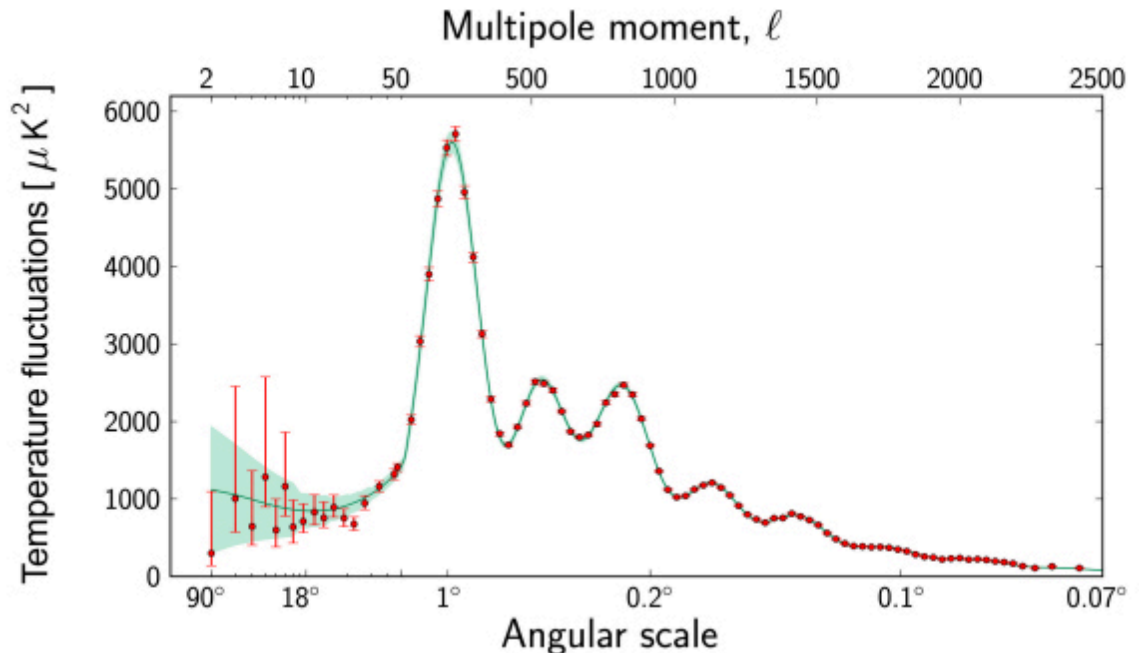


Figure 4. Image from ESA and the Planck Collaboration.

In the CMB temperature power spectrum, the 2nd and 3rd acoustic peaks provide evidence for the ratio of dark matter to baryonic matter [ref]. Dark matter must have been present to provide additional

<sup>11</sup> Vittorio, Silk, 1984

gravitational pull, allowing these small density variations to grow into galaxies and clusters over the course of 14 billion years.

Measurements from space satellite missions such as Planck have mapped these anisotropies in detail, showing that the data fits well with the  $\Lambda$ CDM model. The angular power spectrum displayed a series of acoustic peaks, whose relative heights allude to the total matter density and dark matter content. For instance, Planck 2018 yielded a cold dark matter parameter ( $\Omega_c h^2$ ) of  $0.120 \pm 0.001$ , which was in good agreement with predictions of the  $\Lambda$ CDM model.<sup>12</sup>

CMB provides some of the strongest observational evidence for dark matter. The data not only confirms dark matter's influence on the early universe, but how it shaped the growth of galaxies and large-scale structures over billions of years.

### **E. Structure Formation**

In the  $\Lambda$ CDM model framework, small density fluctuations in the early universe grew under the influence of gravity into the large-scale structures we see today.<sup>11</sup> This "bottom-up" scenario predicts that the earliest structures to collapse were those of small dark matter halos, which later merged to form larger galaxies, groups, and eventually, galaxy clusters.<sup>11</sup>

Observationally, hierarchical formation is supported by a wide range of evidence. High-redshift galaxy surveys (such as HST and JWST) reveal that massive galaxies assemble over time from the merging of smaller objects.<sup>13</sup> The distribution of satellite galaxies across the edges of the Milky Way and Andromeda galaxies are also consistent with the  $\Lambda$ CDM prediction of numerous low-mass subhalos. Additionally, the large-scale distribution of galaxies in the SDSS survey exhibits a filament-like cosmic web structure, which is the type of network expected if small dark matter halos had coalesced into larger ones.<sup>14</sup>

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<sup>12</sup> Aghanim *et al*, 2018

<sup>13</sup> Dolgov, 2023

<sup>14</sup> Alam *et al*, 2018

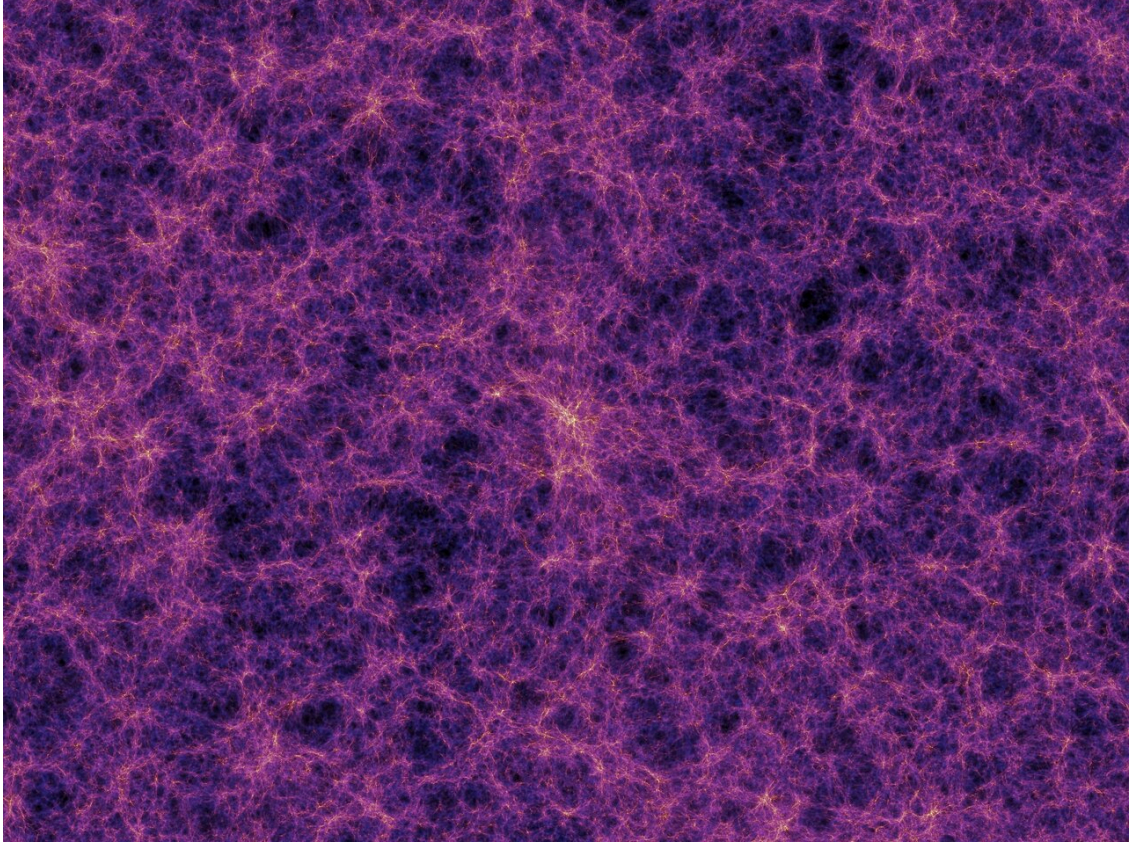


Figure 5. The Cosmic Web. Image from Volker Springel (Max Planck Institute for Astrophysics) *et al.* The image above depicts a simulation of the large-scale structure of the universe, or the cosmic web. The dense filaments and nodes are dominated by dark matter, with galaxies and galaxy clusters tracing these structures. Much of the cosmic web is filled with vast voids, which contain very little matter.

Observations strongly support hierarchical structure formation, in which small dark matter halos collapse first and merge to form larger galaxies and clusters.<sup>15</sup> This bottom-up growth explains the abundance of dwarf galaxies, along with the filament-like assembly of galaxies seen in large surveys. This structure could not be formed by baryonic matter alone; the presence of non-luminous dark matter halos is required in order to provide the scaffolding for the cosmic web.<sup>15</sup>

#### **IV. Dark Matter Particle Candidates**

##### **A. WIMPs**

Weakly Interacting Massive Particles (WIMPs) had long been considered the leading candidate for dark matter because of their natural occurrence in the Standard Model of particle

physics, including supersymmetry.<sup>15</sup> These theoretical particles would not interact through electromagnetism, but through gravity and weak nuclear force, making them undetectable through light-based methods.<sup>16</sup>

Their mass spectrum, generally ranging from a few GeV to several TeV, allows them to be "cold" dark matter, clustering effectively and driving the formation of large-scale structures.<sup>16</sup>

A significant appeal of WIMPs is a phenomenon known as the "WIMP Miracle", where these particles with such weak-scale interactions would have relic abundance from the Big Bang that would naturally match the amount of dark matter observed in the universe.<sup>16</sup>

However, despite numerous experimental efforts over a period of decades, including direct detection, indirect detection, and collider experiments (LHC), WIMPs are yet to be discovered. This has led to the rise of alternate dark matter candidates, drawing attention away from WIMP particles.

## **B. MACHOs**

Massive Compact Halo Objects, or MACHOs, contain very different types of dark matter candidates. Instead of particles, these include astrophysical objects, such as brown dwarfs, black holes, or neutron stars, which emit minimal or no light.<sup>16</sup> Their existence would be indicated through gravitational microlensing, a phenomenon where the light from background stars is temporarily amplified as the MACHO moves in front of them.<sup>17</sup>

Early microlensing surveys (for instance, the MACHO and EROS collaboration), indicated that MACHOs could only account for a small portion of the dark matter halo.<sup>17</sup> Furthermore, they are unable to explain observations such as the cosmic microwave background and large-scale structure, which require non-baryonic dark matter.

While MACHOs may play a role in dark matter composition, especially in the form of primordial black holes, they cannot account for all the dark matter in our universe.

## **C. Ultralight DM (Axions)**

Ultralight dark matter (ULDM) is theoretically composed of extremely light particles, potentially bosons, with masses ranging from  $10^{-22}$  eV to 1 eV.<sup>17</sup> Due to their low masses, they exhibit a wave-like nature on galactic scales.<sup>18</sup>

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<sup>15</sup> Roszkowski *et al*, 2017

<sup>16</sup> Yoo *et al*, 2003

<sup>17</sup> Ferreira, 2020

ULDM could potentially address some discrepancies with the  $\Lambda$ CDM model, including the missing satellites problem and the core/cusp problem. ULDM particles, due to their wave-like nature, suppress the formation of small-scale structures such as dwarf galaxies.<sup>18</sup> This would explain the low number of satellite galaxies orbiting larger galaxies. Additionally, ULDM particles undergo Bose-Einstein condensation, forming a core that replaces the central density cusp predicted by the  $\Lambda$ CDM model.<sup>18</sup>

Axions are also characterized by their low mass, weak interactions, and potential to form Bose-Einstein condensates.<sup>18</sup> They were originally proposed as a solution to the Strong CP problem, as the axion field acts as a dynamic field that can adjust itself to cancel out the CP-violating term in the strong force.<sup>19</sup>

While no axion has been detected yet, ULDM proposes a promising theoretical solution that can be tested in the future.

## V. Conclusion

A large portion of the matter in our universe consists of non-baryonic dark matter, meaning it's essential we continue to study and research its nature. In fields of particle physics, physicists are working on discovering the axion through experiments such as ADMX and HAYSTAC.<sup>19</sup> Additionally, more high-precision measurements of the polarization of CMB from the initial conditions of the universe are being taken, leading to a greater understanding of this mystery.

Dark matter is essential to explaining the structure and evolution of the universe. Observational evidence such as the flat galaxy rotation curves and the collision of two galaxy clusters in the Bullet Cluster provide unignorable reasoning for dark matter's existence. While its nature remains unknown, as we haven't yet directly observed this non-luminous matter, evidence shows that it acts as the framework for galaxies and clusters and is the scaffolding for the cosmic web. Identifying the composition of dark matter will not only solve one of cosmology's greatest mysteries, but transform our knowledge of fundamental physics.

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<sup>18</sup> Adams *et al*, 2022

<sup>19</sup> Dror, 2021

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